HIGHER TECHNOLOGICAL INSTITUTE
Department of Electrical and Computer Engineering

EEC163: Analog Control Systems
First Midterm

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Q1: Consider the linear time invariant system described by the following differential equation:

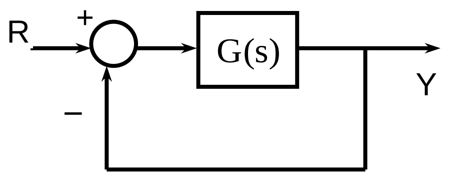
$$\ddot{y}(t) + 3\dot{y}(t) + 2y(t) = \dot{u}(t) + 3u(t)$$

(a) Assume zero initial conditions and calculate system transfer function.

(b) Calculate system unit step response (i.e. u(t) = 1(t)).

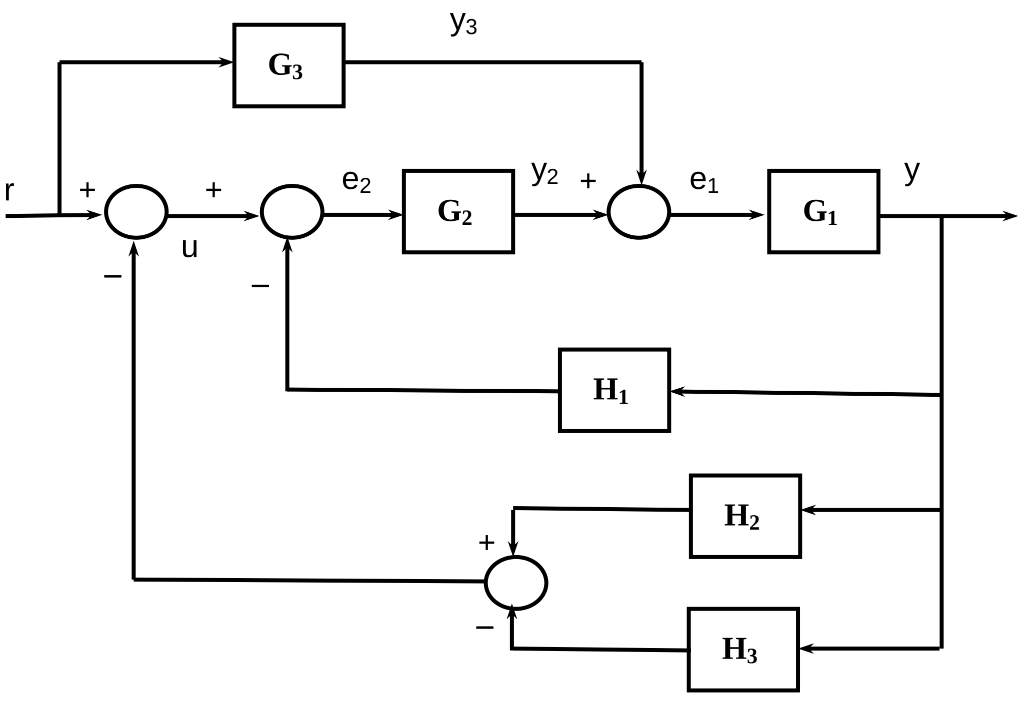
Q2: Consider the unity feedback system with forward gain:

$$G(s) = K\ \frac{2}{s(s+1)(s+2)}$$



(a) Determine system type.
(b) Determine closed loop system steady state error for a unit step input.
(c) Calculate closed loop system steady state error for a unit ramp input for K=10.

Q3: Reduce the block diagram shown below and calculate closed loop transfer function



Answers:

Q1:

(a) Assume zero initial conditions and take Laplace transform of system differential equation to get:

$$(s^2+3s+2)\ Y(s) = (s+3)U(s)$$

and $G(s) = Y(s)/U(s) = \dfrac{s+3}{(s+1)(s+2)}$

(b) Unit step response:

$$Y(s) = \frac{s+3}{s(s+1)(s+2)} = \frac{a}{(s+1)} + \frac{b}{(s+2)} + \frac{c}{s}$$

Where a= –2, b=0.5, and c=1.5

So that $y(t) = \mathcal{L}^{-1}\{Y(s)\} = [1.5-2e^{-t}+0.5\,e^{-2t}]\ \mathbf{1}(t)$

Matlab Code:

```
>> n=[1 3]; d=[1 3 2 0];
>> [r, p, k]=residue(n,d)
    r =
          0.5000
         -2.0000
          1.5000
    p =
         -2
         -1
          0
```

Q2:

(a) System has type 1

(b) For a unit step input, $e_{ss} = 0$

(c) For a unit ramp input, $K_v = \lim_{s\to 0} sG(s) = K$, and for K=10**,** $e_{ss} = 1/K_v = 10\%$

Q3:

$y = G_1 e_1 = G_1 (y_2 + y_3) = G_1 (G_2 e_2 + G_3 r) = G_1 G_2 e_2 + G_1G_3 r$

$= G_1 G_2 (u - H_1 y) + G_1G_3 r = G_1 G_2 u - G_1G_2H_1 + G_1G_3 r$

$= G_1 G_2 (r - H_2 y + H_3 y) - G_1G_2H_1 + G_1G_3 r$

$$\boxed{Y(s) / R(s) = \frac{G_1G_2 + G_1G_3}{1 + G_1G_2H_1 + G_1G_2H_2 - G_1G_2H_3}}$$